

- 1. What is 2021 202 20 + 2?
- 2. A store sells cards in packs of 54. If Alfred wants to get 421 cards, how many packs must be buy?
- 3. How many solutions does the equation |x |2x + 1|| = 3 have?
- 4. Billy has four distinct red cars and three distinct blue cars. How many ways can he choose two cars such that one is red and the other is blue?
- 5. Jim and Terry are trying to fill a 24 liter tank with water. By himself, Jim can fill the tank in 8 hours, while Terry takes an entire day to do it. How many hours does it take for both of them to fill the tank?
- 6. Alex has two coupons for buying skins on Fortnite. One coupon decreases the price by 100 V-bucks, and the other decreases the price by 10%. These two coupons can be used at the same time. If Alex wants to buy the *Skull Trooper* skin, which usually costs 1500 V-bucks, what is the least amount of V-bucks he can pay to get it?
- 7. A Pringles jar contains some amount of marbles. Jim takes half of the marbles out of a Pringles jar that was 1/3 full. If Jimmy has 2 marbles, how many marbles can fill a Pringles jar to the brim?
- 8. The average of Jahseh's ten test scores is 96. If all of his test scores are between 0 and 100 inclusive, what is the minimum possible score he could have gotten on one of his tests?
- 9. What is the area of a triangle with side lengths 20, 21, 29?
- 10. In a group of 30 students, 10 students like math, 10 students like English, and 10 students like science. No students like English and math, while four students like math and science. In addition, one student likes science and English. How many students do not like all three subjects?
- 11. A *skirt-like* word is a string of 5 distinct letters that is different from the word "skirt" in at most one position. For example, "skirt", "shirt", and "akirt" are *skirt-like*, but "skrrf" and "smart" are not *skirt-like*. How many *skirt-like* words are there?
- 12. Andrew, Binhan, Mingzhen, Dylan, and Eric sit down in 5 chairs labeled 1-5 from left to right to take a group picture. Andrew wants to either be in chair 1 or chair 2. Binhan wants to either be in chair 2 or chair 3. Mingzhen wants to either be in chair 3 or chair 4. Dylan wants to either be in chair 4 or chair 5. Eric does not care what chair he ends up in. How many ways can they line up to take a photo?
- 13. The numbers 1 through 25 are written in a 5 by 5 chessboard. If any two consecutive integers are adjacent on the chessboard, what is the sum of the numbers on the black squares given that the corners are black? Adjacent squares are squares that share a side.

## 2021 New Year's Invitational Sprint Round



- 14. Pranav and Anivrit are playing a game. Pranav chooses a secret number which will be an integer from 0 to 2020 inclusive, while Anivrit guesses what the number is. Every time Anivrit guesses correctly, Pranav changes his secret number to a larger integer from 0 to 2020, but does not tell Anivrit. Anivrit wins once Pranav cannot change his secret number anymore. What is the least number of guesses Anivrit needs to guarantee he wins, no matter how Pranav chooses his secret number(s)?
- 15. All of the altitudes of triangle ABC have integer lengths. If two of the altitudes have length 30 and 15, what is the minimum possible length of the last altitude?
- 16. Bobby picks some integers from 1 through 20, inclusive. If they are all pairwise relatively prime, what is the maximum sum of the numbers?
- 17. If a and b are distinct numbers such that  $a^2 = b + 10$  and  $b^2 = a + 10$ , what is |ab|?
- 18. What is the sum of all possible values of gcd(4x+y,4y+x) when x and y are relatively prime positive integers?
- 19. For an integer n, let f(n) be the greatest positive integer k such that n is divisible by the product of the smallest k prime numbers. For example, f(1) = 0, f(2) = 1, f(3) = 0, and f(6) = 2. What is  $f(1) + f(2) + f(3) \dots + f(2020)$ ?
- 20. A polynomial P(x) with degree 4 satisfies P(0) = 1, P(1) = 3, P(2) = 9, P(3) = 27, P(4) = 81. What is P(5)?